Radon priority areas and radon extremes
- an initial study -

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Content

• Rationale:
  - Importance of indoor radon
  - Legal background
• Definitions of Radon priority areas
• Extremes, anomalies, outliers: Causes and statistics
• How to detect and to model anomalies?
• Examples
Very short introduction
Indoor radon – most important contribution to dose!
Second most important cause of lung cancer after smoking!
In Europe estimated about 62,000 lung cancer fatalities per year attributed to Rn.
(incl. RU, TR; missing: BiH, LV, MD, MK, MT, RS, UA)

Japan: estimated ca. 3,100 (Gaskin et al., Envir. Health Perspectives 125, 5 (2018))

Sources of indoor Rn:
1. Geogenic Rn (most important in most cases)
   2. Building materials
   3. Tap water, natural gas

Concentrations of indoor Rn controlled by
Geogenic factors:
   Geology, soil type, U concentration in topsoil, permeability, granulometry,…
Anthropogenic factors:
   Construction type (tightness of structures in contact with the ground),
   life or usage patterns (ventilation)

Very high local and temporal variability → makes prediction very difficult.
Basic Safety Standards (BSS)

Council Directive 2013/59/Euratom laying down basic safety standards for protection against the dangers arising from exposure to ionizing radiation


Art. 103,3; RPA:
“Member States shall identify areas where the radon concentration (as an annual average) in a significant number of buildings is expected to exceed the relevant national reference level.”

*Conceptual* definition, which has to be translated into an *operable* definition.

Art. 54, 74, annex XVIII; Radon Action Plan:
In areas according Art.103,3: Buildings with public access and workplaces must be measured and if above RL, remediated. New buildings: particular Rn prevention. Strategy to reduce Rn in dwellings.

Reference level (RL): must be ≤ 300 Bq/m³ (BSS Art 54,1 & 74,1). Most countries chose 300, Ireland and others: 200

These areas are called **Radon Priority Areas (RPA)**, to indicate priority in taking action. Formerly also “Radon Prone Areas”
RPA definitions, 1

Some examples of operable RPA definitions, based on different Rn measures:

- An area B (grid cell, municipality...), in which the mean population-weighted indoor concentration C exceeds the reference level (RL); $AM_B(C) > RL$; measure = $AM_B$

- same, but indoor concentration in *dwellings on ground floor*

- An area B, in which the probability that C exceeds the RL, is greater than p (typically 10%); $prob_B(C > RL) > p$; measure = $prob_B$

- The areas B which represent the upper 10% of $AM_B(C)$; measure = percentile

- An area, in which the collective exposure (e.g., $AM_B(C) \times$ population) is among the upper 10%

**Important:**
There is no “natural” definition of RPA! Therefore, also no “true” RPA! RPAs always depend on definition and to some extent, on estimation method. This is partly a political decision, partly a pragmatic one (i.e., availability of data).

**Consequence:**
RPAs may, in general, not be comparable across borders. This may create communication and credibility problems. Discussing this and proposing solutions is another subject of the Metro Radon project. One way may be a map of the Rn hazard index (RHI – currently under development) as “universal” (but still to an extent deliberate) measure of Rn “prioritiness”.
RPA definitions, 2

Multinominal:

Instead of 2 classes (RPA / non-RPA), several classes of “Rn-priorityness”; approach chosen by some countries.

Multivariate:

Although the BSS definition relies on indoor Rn concentration, one may chose to base estimation on other Rn-related variables instead or additionally. Examples: geogenic Rn potential, U concentration in the ground, terrestrial gamma dose rate, geological unit, tectonic features etc.
The problem

- Rn concentrations are spatially very variable. Reason: the variability of the physical factors which cause Rn concentration.
- Therefore, also in an area labelled non-RPA, i.e. “harmless”, small sub-areas can still have high Rn concentrations.
- Occurrence probability of small extremal areas should be quantified – possibly a complementary RPA criterion.
- Usual RPA criteria (last 2 slides) cover “most” of an area, but also houses in small “anomalous” areas deserve attention!

The task

Find indicators of the presence of spatially small extremes
- from data
- via models
- from physical predictors
An artificial example
background and anomalies, 1

![Graph showing Rn concentration across different areas with annotations for active fault, granite, RPA, and RPA threshold.](image)

- "true" Rn concentration
- Granite
- Active fault
- RPA, if AM(C in area) > RL
- RPA
- Mean
- RL
- Area 1, Area 2, Area 3, Area 4, Area 5, Area 6
background and anomalies, 2

RPA, if $Q_{95}(C \text{ in area}) > RL$

RPA, if $Q_{99}(C \text{ in area}) > RL$
RPA, if $p = \text{prob}(C > RL \text{ in area}) > p_0$

RPA, if in area anomalies $> RL$ are present
Some concepts
& little bit physics
Physical causes of small anomalies

- **Geological** anomalies, e.g. small granite outcrops
- **Secondary U mineralization:** hydrothermal activity, contact metamorphism,...
- Active **faults**
- **Karst** formations (high macro-permeability)
- Anthropogenic: **mining** residues,...
Questions:
How can one estimate statistics from limited and uncertain sample values?

• AM is known to be sensitive against outliers
• High quantiles or exceedance probabilities of high RL are difficult to estimate from data
• “Rare” events are likely be missed
• Opposite: Outliers can represent “spurious” anomalies
• After all, what is an “anomaly”?
What is an anomaly?

- Concepts of “anomaly” and “background” cannot be separated! Definition of anomaly depends on definition of BG and vice versa.

- Qualitatively:
  Anomaly is a value which is significantly different from its vicinity.

- Identifying anomalies:
  - residuals from interpolated smooth surface
  - high difference from neighbours
  - assuming a distribution within a vicinity → outliers of the distribution
  - high Hölder exponent $\alpha$
    
    from fractal theory: $z(A) \sim |A|^\alpha$ ($z(A) = \text{mean over } A$, $|A| = \text{area of } A$) $\rightarrow \alpha = \lim_{|A| \rightarrow 0} \log z(A)/\log |A|$

Do not confuse an anomaly with an extreme of the background! Anomalies and BG extremes represent different (physical and statistical) processes!
**Terminology**

**Outlier:** a value which one suspects not belonging to an assumed population.
   Reasons can be:
   - observation error or uncertainty
   - an accidentally isolated extreme of the population
   - an instance which belongs to a different population

**Extreme:** The highest or lowest value of a set. Does not say anything about its nature.

**Anomaly:** a point or a region, at which the variable behaves differently from most of the data domain or of its neighbourhood, or simply being “too large” (or too small) than is considered typical for that part of the field.
   An anomaly may be an outlier, but does not need to be one necessarily.

**Hot spot:** seems to be mostly used for points, or cluster of points, or small regions, where the variable takes anomalously high values.
   Reasons can be:
   - A region in which the background process takes high levels
   - A region which is the domain of a separate process.
   Distinguishing these can be quite complicated!

Anomaly and hot spot: mostly seem to denote “true” effects, i.e. not related to observation; “outlier” seems to be neutral in this respect, i.e. can also denote observation effects... but terminology is not completely unambiguous!
Some mathematics
Often data are not sufficient for estimating occurrence of anomalous events.

On the other hand, establishing models from which statistics are derived, can also be sensitive to uncertainty.

**Popular model for Rn:** Lognormal distribution (LN) – Requires either

- estimating geometrical mean (GM) and standard deviation (GSD) from data, GM=exp(AM(ln Z)), GSD=exp(SD(ln Z)); or

- establish QQ-plot with empirical quantiles and determine GM and GSD by regression.
  This also allows checking whether LN is (approximately) valid.

Then, quantile $Q_p = \exp(\Phi^{-1}(p | GM, GSD))$

exceedance probability: $\text{prob}(Z > z) = \Phi((\ln GM - \ln z)/\ln GSD)$

$\Phi$ = standard normal, more precisely to be replaced by $t_{n-1}$, $n$=sample size.

**Big problem:** One cannot assume LN for anomalies on top of background!

Establishing a BG model is relatively (!) easy. This helps us determining extremes of the BG process, but says nothing about the anomaly process.
Let $Z$ a random variable $\sim LN$ with GM=1 and GSD=2. Take $n$ independent random samples $\{z_1, \ldots, z_n\}$. Which is the maximum value of these?

Graph (based on 100,000 simulations):
- Expected maximum $E(z_{\text{max}})$,
- 90% confidence interval ($Q05 – Q95$)

If we have a sample of size $n$ and use a model for the BG (such as LN), and if we find a value $z^\#$ which is above the expected maximum, $z^\# > E(z_{\text{max}})$ or $> Q95(z_{\text{max}}) \Rightarrow$ candidate for anomaly!
distribution of maxima, 2

The maximum $z_{\text{max}} = y_n := \max\{z_1, \ldots, z_n\}$ is exactly distributed $\sim F_Z(y_n)^n$ for $Z \sim F_Z$; but difficult to handle

From extreme value theory it is known that for $F=\text{LN}$ or similar, $y_n$ is asymptotically distributed $\sim \exp(-\exp(-(y_n - \mu_n)/\sigma_n))$ (Gumbel or EV type1 distribution ... Fisher–Tippett–Gnedenko theorem), mean $= \mu_n + \gamma \sigma_n$, SD=$\sigma_n \pi/\sqrt{6}$, $\gamma=\text{Euler-Mascheroni constant} = 0.57722...$; $\mu_n$ and $\sigma_n$ are (complicated) functions of parameters of $F_Z$ and sample size $n$. However, converges slowly; better: fit simulated by power function (in the above example $r^2>0.995$)

pdf of maxima of LN(0,ln(2)) for different sample sizes

$Q_p$ easy to calculate: $Q_p(Y_n)=F_{Z^{-1}}(p^{1/n})$
Dilemma!

If we estimate directly from data, i.e. without model assumptions (such as LN):
perhaps we miss important but spatially small phenomena because of insufficient amount of data;

If we use a model, derived from data and possibly additional knowledge (in Bayesian statistics: prior), and define anomalies as deviations from the model:
- the choice of model assumptions may be wrong and
- its parameterization may be inaccurate.

Possible pragmatic way out?

Use all available approaches together and decide from “expert knowledge”:
• Inspect data and derive empirical statistics;
• Use a BG model and look for residuals and outliers;
• Look for physical indications: certain geological (lithology and hydrogeology) and tectonic features.

But the BG model alone will not inform us about occurrence and size of anomalies ... limit of modelling!
Real examples!
Example 1: Hirosaki region

red: felsic (acid) plutonic; yellow: felsic volcanic

Source: Geological Survey of Japan
Example 2: European indoor Rn map

**Empirical mean**

AM(indoor Rn concentration) in 10 km x 10 km cells; ground floor rooms

ca. 28,000 non-empty cells,
ca. 1.1 mill data, 34 countries
(by Aug 2018)

From European Atlas of Natural Radiation:

**Estimated high quantile**

$$\widetilde{Q}_p = \exp \left( t_{p; n-1} SDL + AML \right)$$

n=number of data in cell
AML:=AM(ln z)
SDL:=SD(ln z)

LN model with observed AML, SDL assumed!
Example, cont.

**Estimated exceedance probability**

\[ p = \text{prob}_B(Z > z); \quad \hat{p} = t_{n-1} \left( \frac{1}{\sqrt{n+1}} \right); \quad \zeta := \frac{\ln z - \text{AM}_B(\ln Z)}{\text{SD}_B(\ln Z)} \]

LN model with observed AML, SDL assumed!

**Estimated relative excess probability**

Probability that \( k \times \text{median} \) is exceeded, relative to median

\[ \varepsilon_k = \frac{\text{prob}(Z > k \cdot \text{Med})}{\text{prob}(Z > \text{Med})} \]

\[ \varepsilon_k = 2 \left[ 1 - t_{n-1} \left( \frac{\ln k}{\sqrt{n+1} \cdot \text{SDL}} \right) \right] \]

LN model with observed SDL (only) assumed!
Example, cont.: fractal indicators

\( \eta := \frac{z(\text{cell})}{AM(z, \text{neighbouring cells})} \)

Q99(\( \eta \))=4.35

\( \alpha \approx -\lim_{|A| \to 0} \frac{\log z(A)}{\log |A|} \)

estimated by defining A as shells of different radius around a cell, then regression

Q99(\( \alpha \))=0.194

No model assumed! Only from observed data
Example, cont.: maxima

Expected maximum, assuming LN (from data) and given sample size n

Additional problem, not to be discussed here: Spatial interpolation of extremes for generating “map of extremes” is mathematically demanding.

Ratio between observed maximum and estimated 95% quantile.
If ratio > 1:
- indicator for anomaly
- or: distribution assumed wrongly

Data errors?
Finally...
Conclusions & To-do

• RPA definition and estimation: not only academic exercise, but practically important. May have severe economic & political impact. Heavy stakeholder interest! Therefore: QA very important!

• RPAs can be very small in space, due to small geological or tectonic features. They may not contribute much to the overall mean, but may still be locally relevant.

• Presented here: some initial ideas....

• Often not or insufficiently covered by data - Identification, quantification and modelling not easy mathematically!

• To do:
  - Further develop mathematical tools
  - Explore validation possibilities
  - Uncertainty in estimation of “small” but extreme phenomena
  - Once anomaly quantification achieved: define RPA criterion
  - Develop a generic strategy to deal with Rn anomalies!

RPA – a sensitive subject!
Action required in RPA can be costly → political disputes
Thank you!

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