

1. Introduction. The objective of this work is to present a method for characterization of the light absorbing properties of 3D printed mesh filters used for efficiency variation in the Triple-to-Double Coincidence Ratio (TDCR) method in liquid scintillation counting. The characterization allows a priori estimation of the detection efficiency and TDCR value that will be obtained when the sample with a filter is measured.

2. Methods and Materials. For the application of the TDCR method special 3-PMT detectors are required (Figure 1) that count double (AB, BC, AC), triple (T) and logical sum of double (D) coincidences. According to the TDCR model the ratio of the triple to the logical sum of double coincidences counting efficiency in a detector with 3 identical photomultipliers is expressed as [1]:

$$\frac{\varepsilon_T}{\varepsilon_D} = \frac{\int_0^{E_{max}} S(E) \left(1 - e^{-EQ(E)/3\lambda}\right)^3 dE}{\int_0^{E_{max}} S(E) \left[3 \left(1 - e^{-EQ(E)/3\lambda}\right)^2 - 2 \left(1 - e^{-EQ(E)/3\lambda}\right)^3\right] dE} \quad (1)$$

where λ is the free parameter of the model, hereafter called "figure of merit". **The figure of merit is defined as the number of photoelectrons created at the photocatode of the phototube per keV energy released in the cocktail.** $Q(E)$ is the ionization quenching correction factor which is calculated using the Birks semi empirical formula [3]:

$$Q(E) = \frac{1}{E} \int_0^E \frac{dE}{1 + kB(dE/dx)} \quad (2)$$

The parameter kB is the ionization quenching parameter measured in units cm/MeV. The kB parameter is external for the TDCR model, but its value can be obtained by varying the detection efficiency of the detector by placing filters over the sample. If the correct kB value is used in the model, the calculated activity of the sample should be independent of the detection efficiency [2].

The figure of merit λ can be determined from the ratio of the triple to the logical sum of double events TDCR which converges to the ratio of efficiencies $\varepsilon_T/\varepsilon_D$.

Figure 1. Schematic of a 3-PMT TDCR counter

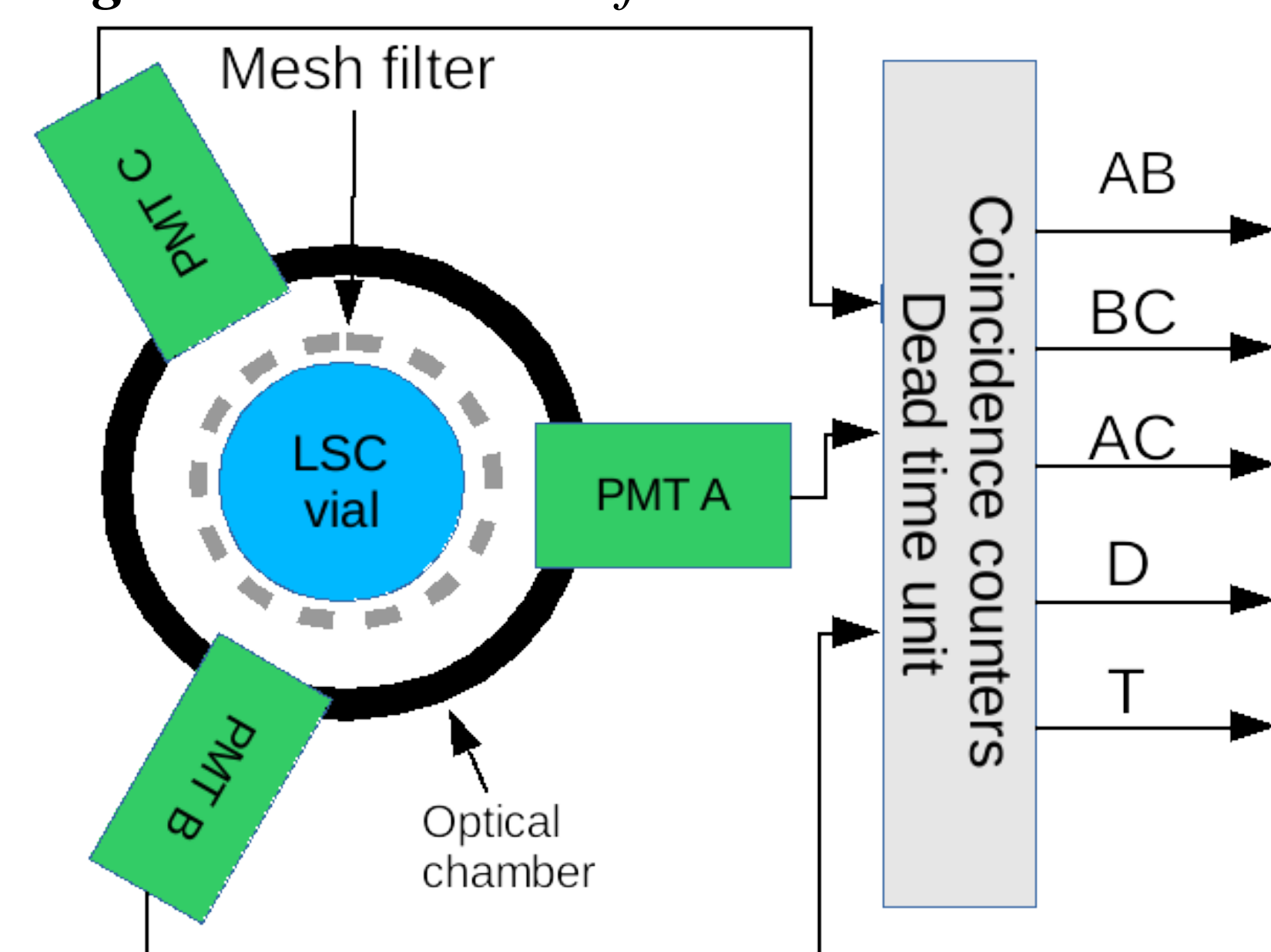
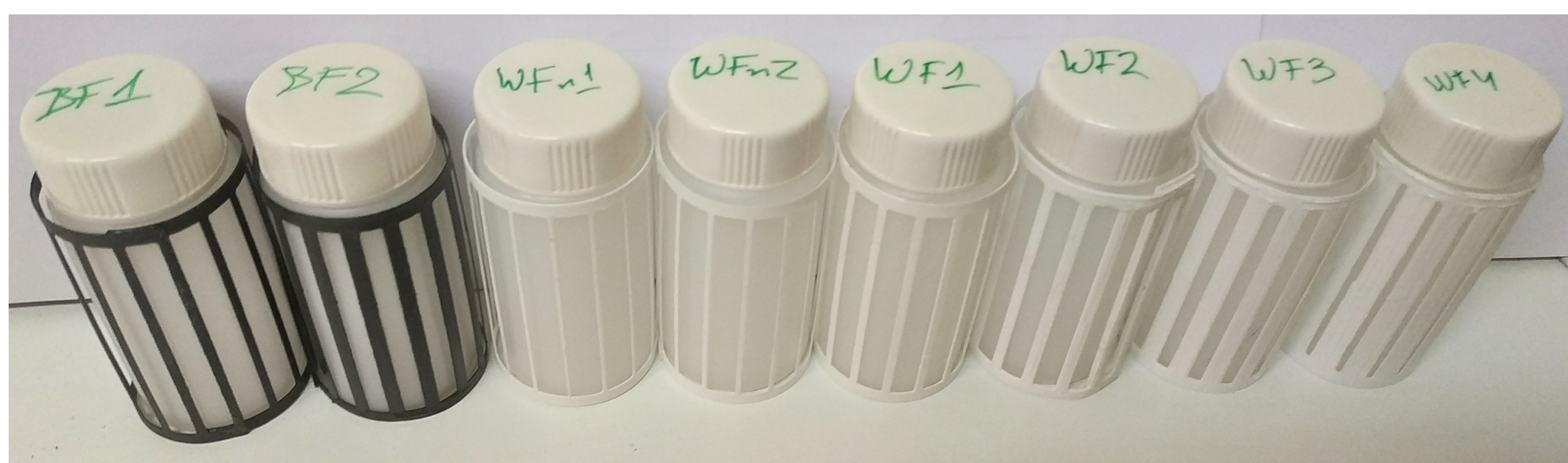


Table 1. R-values of the 3D-printed filters

| Filter Name | PLA color | λ_i , e ⁻ /keV | R |
|-------------|-----------|-----------------------------------|------------|
| None | - | 1.0601(6) | - |
| WFn1 | White | 1.0003(10) | 0.9436(11) |
| WFn2 | White | 0.9229(10) | 0.9229(10) |
| WF1 | White | 0.9168(10) | 0.8649(10) |
| WF2 | White | 0.8546(10) | 0.8062(10) |
| WF3 | White | 0.7063(11) | 0.7337(7) |
| WF4 | White | 0.6754(10) | 0.6371(10) |
| BF1 | Black | 0.5642(5) | 0.5322(10) |
| BF2 | Black | 0.4140(5) | 0.3906(10) |

Figure 2. 3D printed mesh filters, from left to right BF1, BF2, WFn1, WFn2, WF1, WF2, WF3, WF4



3. Experimental Studies. The proposed 3D printed cylindrical mesh filters for the realization of the efficiency variation technique (Figure 2) were made on a 3D printer using white or black PLA (a common polymer in 3D printing).

For the characterization of the 3D printed mesh filters we propose the ratio:

$$R = \frac{\lambda_i}{\lambda_0} = \frac{\text{Photoelectrons per keV with a filter}}{\text{Photoelectrons per keV without a filter}} \quad (3)$$

where λ_i and λ_0 are the figures of merit with and without filter respectively.

The value of R should be intrinsic to the filter and independent from the measured sample. To determine R for each filter, the figure of merit λ has to be determined from the TDCR model for measurements of a source with and without a filter.

The R values of the set of 3D printed mesh filters (Table 1) were determined with a ³H in toluene source and the TDCR07c code [4].

An experimental study was performed with the TDCR-SU detector system at Sofia University [5] to test the predictive abilities of the R value. The measured samples were ³H in UltimaGold ($\lambda_0 = 0.6019$ e-/keV) and ¹⁴C in toluene ($\lambda_0 = 0.8593$ e-/keV). Equation (3) was used to determine the expected figure of merit with applied filters from the measurement without a filter. The predicted TDCR is calculated from λ_i using the TDCR07c code.

The predicted and measured TDCRs for ³H in UltimaGold (Table 2) and ¹⁴C in toluene (Table 3) agree well for most measurements within 0.7%.

Table 2. Predicted TDCR for a ³H source, using R-values of the 3D-printed filters.

| Filter | R | λ_i , e ⁻ /keV | TDCR (predicted) | TDCR (measured) | Diff, % |
|--------|------------|-----------------------------------|------------------|-----------------|---------|
| None | 1.0000 | 0.6019 | - | 0.3750(12) | - |
| WFn1 | 0.9436(11) | 0.5765 | 0.3597(30) | 0.3604(14) | -0.19 |
| WFn2 | 0.9229(10) | 0.5638 | 0.3536(29) | 0.3524(15) | 0.34 |
| WF1 | 0.8649(10) | 0.5283 | 0.3361(28) | 0.3340(12) | 0.63 |
| WF2 | 0.8062(10) | 0.4925 | 0.3179(26) | 0.3206(12) | -0.66 |
| WF3 | 0.7337(7) | 0.4482 | 0.2943(24) | 0.3000(16) | -1.90 |
| WF4 | 0.6371(10) | 0.3892 | 0.2613(22) | 0.2630(13) | -0.65 |

Table 3. Predicted TDCR for a ¹⁴C source, using R-values of the 3D-printed filters. The value in bold was used to determine all other figures of merit.

| Filter | R | λ_i , e ⁻ /keV | TDCR (predicted) | TDCR (measured) | Diff, % |
|--------|------------|-----------------------------------|------------------|-----------------|---------|
| None | 1.0000 | 0.8593 | - | 0.9482(2) | - |
| WFn1 | 0.9436(11) | 0.8109 | 0.9452(34) | 0.9436(2) | 0.17 |
| WFn2 | 0.9229(10) | 0.7931 | 0.9440(34) | 0.9427(2) | 0.14 |
| WF1 | 0.8649(10) | 0.7432 | 0.9403(34) | 0.9385(2) | 0.20 |
| WF2 | 0.8062(10) | 0.6305 | 0.9298(34) | 0.9280(3) | 0.20 |
| BF1 | 0.5322(10) | 0.4573 | 0.9031(35) | 0.8986(3) | 0.50 |
| BF2 | 0.3906(10) | 0.3365 | 0.8674(35) | 0.8623(4) | 0.59 |

4. Conclusions. The R value, defined as the ratio of the figure of merit with filter to the figure of merit without a filter, seems to be independent of the measured cocktail and radionuclide. A good agreement can be observed between the predicted TDCR values and the experimentally obtained TDCR values for the measurement of ³H in UltimaGold and ¹⁴C in toluene samples. **The R value can be used to identify unstable cocktails as well as for choosing proper filters from a set of many for applying the efficiency variation technique.**

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6. References.

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